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LATTICE OF NEAR-RING CONGRUENCES ON SEMINEARRINGS

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Abstract. The bijections (i) between the set of all near-ring congruences on a seminearring S and the set of all generalised strong dense reflexive right k-ideals and (ii) between the set of all zero-symmetric near-ring congruences on a seminearring S and the set of all generalised strong dense reflexive k-ideals established in Theorems 5 and 6 (Chakraborty, Mukherjee and Sardar, 2022) (*cf.* Note 2,6), respectively, have been extended to lattice isomorphisms by putting some restrictions on S. A detailed study of modularity, distributivity and completeness of these lattices has also been accomplished.

Key Words and Phrases: near-ring congruence, zero-symmetric near-ring congruence, strong ideal, strong dense ideal, generalised strong dense reflexive (right) k-ideal.

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1. Introduction. According to G. Pilz (1977), a *near-ring* is a non-empty set N together with two binary operations '$+$' and '$\cdot$' such that (i) $(N, +)$ is a group, (ii) $(N, \cdot)$ is a semigroup and (iii) for all $n_1, n_2, n_3 \in N$, $(n_1 + n_2) \cdot n_3 = n_1 \cdot n_3 + n_2 \cdot n_3$, *i.e.*, '$\cdot$' distributes over '$+$' from the right side ("right distributive law"). Let G be an additive group (not necessarily abelian). Then the set $M(G)$ of all mappings from G into G forms a near-ring under point-wise addition and composition of mappings (Pilz,1977). If we consider an additive semigroup S (not necessarily commutative) instead of the group G, then the set $M(S)$ of all self-maps of S becomes an algebraic structure what is known as seminearring. To be specific, a *seminearring* $(S, +, \cdot)$ is an algebraic structure such that $(S, +)$ is a semigroup, $(S, \cdot)$ is a semigroup and '$\cdot$' distributes over '$+$' from one side *i.e.*, either from the left or from the right accordingly it is called a left distributive seminearring or a right distributive seminearring. Thus seminearrings not only generalize semirings but also near-rings as well as rings. Throughout our work '*seminearring*' stands for *right distributive seminearring*. A seminearring $(S, +, \cdot)$ is said to be *with zero* (0) if 0 is the identity element of $(S, +)$ and 0 satisfies the property $0 \cdot a = 0$ for all $a \in S$. A seminearring S is said to be *zero-symmetric* if S is with 0 such that $s \cdot 0 = 0$ for all $s \in S$.

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